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DEPARTMENT OF MATHEMATICS

SEM – I (PAPER II)

TOPIC: De-moivers theorem

PRESENTED BY

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De Moivre's Theorem

Statement

- Whatever may be the value of n (+ve,-ve integers

  or fraction) the value of) $(\cos\theta + i \sin\theta)^n$ is $\cos n\theta$

  $+ i\sin n\theta$

- $(\cos\theta + i \sin\theta)^n$ is $\cos n\theta + i\sin n\theta$ for all n belongs to I

We have to prove that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i \sin n\theta$$

Now we consider three different cases

Case 1

N= positive integers

We prove that the theorem by the principle of induction

Denote

- $P(n) = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ ____ 1
- Put n = 1 in eqn _______ 1
- $P(1) = (\cos\theta + i\sin\theta)^1$
- $(\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta$
- Thus p(n) is true for n=1 assume that p(n) is true for n=k i.e $p(K) = (\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$___________(2)

- Now we prove that eqn 1 is true for n=k+1 also

- Now multiplying eqn 2 by (cos θ + isin θ) to both sides

- $(\cos\theta + i \sin\theta)^k (\cos \theta + i \sin\theta) = (\cos k \theta + i \sin k \theta) (\cos \theta + i \sin\theta)$

- $(\cos\theta + i \sin\theta)^{k+1} = \cos k\,\theta \cos\theta + i\cos k\,\theta.\sin\theta + i\sin k\,\theta.\cos\theta - \sin\theta.\sin k\,\theta$

- $(\cos\theta + i \sin\theta)^{k+1} = \cos (k + 1)\,\theta + i \sin (k + 1)\,\theta$

- Cos (a+b) = cos A .cos B – sin A.sin B

- sin (A+B) = cos A.sin B + sin A.cos B

- Thus, $(\cos\theta +. i \sin\theta)^{k+1} = \cos (k + 1) \theta + i \sin (k + 1) \theta$

- Then p (k+1) is true

- Hence 1 p(1) is true

- P(k) is true = p(k+1) is also true

- This implies p (n) is true for all n

Case 2

- When n is –ve integer
- Let n be a –ve integer denote n=-m (m is+ve)
- Now, $(\cos\theta + i\sin\theta)^n = (\cos\theta + i\sin\theta)^{-m}$
- $= 1/(\cos\theta + i\sin\theta)^m$

$= 1/(\cos m\theta + i\sin m\theta)$

(by case 1, as m is +ve)

Now we express RHS of eqn 1 as x+iy. For this multiply above and below by the conjugate of the denominater i.e., $\cos m\theta - i\sin m\theta$

- Eqn 1 gives,

$(\cos\theta+i\sin\theta)^n$

$=1/(\cos m\theta+i\sin m\theta)*\cos m\theta-i\sin m\theta/\cos m\theta-i\sin m\theta$

$=\cos m\theta-i\sin m\theta$

$=\cos(-m\theta)+i\sin(-m\theta)$ (since $\cos\theta=\cos(-\theta)$

$\sin(-\theta)=-\sin\theta$)

$(\cos\theta+i\sin\theta)^n=(\cos n\theta+i\sin n\theta)$ (since $n=-m$)

Thus DMT is true for n=-ve integer.

Case 3.

n= fraction =p/q where $q \neq 0$ and +ve or-ve

By case 1, as q is +ve integers

$(\cos \theta/q + i\sin \theta/q)^q = \cos(\theta/q \,.q) + i\sin(\theta/q \,.q)$

$(\cos \theta/q + i\sin \theta/q)^q = \cos \theta + i\sin \theta$-------------1

Taking power 1/q to both side

$(\cos \theta/q + i\sin \theta/q) = (\cos \theta + i\sin \theta)^{1/q}$

$\Rightarrow \cos(\theta/q) + i\sin(\theta/q)$ is one of the value of

$(\cos \theta + i\sin \theta)^{1/q}$ i.e., one of the value of

$(\cos\theta + i\sin\theta)^{1/q}$ is $\cos\theta/q + i\sin\theta/q$

Now raising both sides to the power p

=>one of the values of $(\cos\theta + i\sin\theta)^{p/q}$ is

$\cos(p/q \cdot \theta) + i\sin(p/q \cdot \theta)$------by case 2

Hence one of the value of $(\cos\theta + i\sin\theta)^n$ is

$\cos n\theta + i\sin n\theta$.

Thus DMT is proved when n is fraction.

Applications

- **To compute nth root of complex number.**

THANK YOU